Note on j Factors for Turbulent Flow in Annuli

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In 1933 Colburn (*1*) presented an empirical heat transfer-momentum transfer analogy by which j factors for heat transfer could be determined from the friction factor by the relation

$$j_H = f/2 \tag{1}$$

where $j_H = (h/C_pG)(C_p\mu/k)^{2/3}$ and is called the heat transfer j factor. This analogy has been reasonably successful in predicting turbulent flow heat transfer coefficients provided that the friction factor in Equation (1) is a direct measure of the shear stress at the wall where the coefficient is to be obtained. For example with turbulent flow in circular tubes Equation (1) is reliable since the fanning friction factor is defined as

$$f = \frac{\tau_w \rho \, U^2}{2\, g_c} \tag{2}$$

On the other hand Equation (1) is not suitable for noncircular ducts since the wall shear stress is variable over the wall at a given cross section, and the friction factor as usually defined represents an average of the wall stress. The situation is somewhat similar with concentric annuli in that the shear stress, although symmetrical about the axis of the annulus, is different at the inner and outer wall.

Several heat transfer-momentum transfer analogies have been proposed (*3, 6*), and although they are more theoretical than the Colburn analogy they also depend on the friction factor which must be directly related to the shear stress at the solid boundary. Many of these analogies are applicable to fluids of low Prandtl number, where the Colburn analogy is useful for fluids with a Prantdl number greater than 0.6.

This paper presents a method by which the Colburn analogy (and other analogies which are based on the friction factor) may be used to predict heat transfer j factors for the inner and outer wall of concentric annuli. The relationships presented may be applied to annuli of any diameter ratio from circular tubes to parallel planes.

Consider steady fully developed turbulent flow (no entrance effects) in a concentric annulus made up of tubes of radii r_1 and r_2 (see Figure 1). The turbulent velocity profile is not symmetrical in the annular gap, but the point of maximum velocity r_{max} is displaced toward the axis. A force balance on an element of annular fluid shows that for steady flow the shear stresses at the inner and outer walls of the annular duct are related as follows:

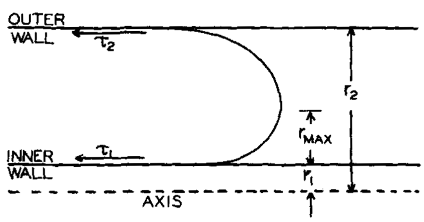

Fig. 1. Cross section of concentric annulus.

$$\frac{\tau_1}{\tau_2} = \frac{r_2(r^2_{max} - r_1^2)}{r_1(r^2_2 - r^2_{max})} = \frac{\lambda^2 - a^2}{a(1-\lambda^2)} \tag{3}$$

where $a = r_1/r_2$ and $\lambda = r_{max}/r_2$.

It has been shown experimentally (*9*) that for turbulent flow

$$r^2_{max} = \frac{r_2^2 - r_1^2}{\ln\,(r^2_2/r_1)^2}$$

or, in dimensionless form

$$\lambda^2 = -\frac{1-a^2}{\ln a^2} \tag{4}$$

This position of the maximum velocity for turbulent flow is the same as the theoretically derived position for laminar flow (*3*).

It has been common practice to base the friction factor and Reynolds number for annuli (and noncircular ducts) on the equivalent diameter defined as four times the cross-sectional area divided by the wetted perimeter. Thus for annuli

$$N_{Re} = \frac{2(r_2 - r_1)U}{\nu} = \frac{2\,b\,U}{\nu} = \frac{(1-a)2r_2U}{\nu} \tag{5}$$

and

$$-\frac{1}{\rho}\frac{dP_f}{dx} = \frac{2\,f\,U^2}{2g_c(r_2 - r_1)} = \frac{2\,f\,U^2}{2g_c\,r_2(1-a)} \tag{6}$$

On the basis of these definitions the friction factor-Reynolds number relationship for annuli is approximately the same as for circular tubes. Empirical correlations of data however have also included some function of the radius ratio (*2*). The friction factor as defined above bears no direct relation [in the form of Equation (2)] to the shear stress at either wall of the annulus, and therefore j factors for either wall cannot be obtained from the friction factor by Equation (1).

Clearly some expression is needed to directly predict the shear stress existing at each wall. Rothfus, et al. (*9*) defined two friction factors for annuli as follows:

$$f_1 = \frac{\tau_1\,\rho\,U^2}{2\,g_c} \tag{7}$$

$$f_2 = \frac{\tau_2\rho\,U^2}{2\,g_c} \tag{8}$$

These workers determined that

$$\frac{1}{\sqrt{f_2}} = 4.0 \log\,[(N_{Re})_2\sqrt{f_2}] - 0.40 \tag{9}$$

where the Reynolds number $(N_{Re})_2$ is based on the equivalent diameter of the outer portion of the annular stream in the region $r_2 \geq r \geq r_{max}$. Hence

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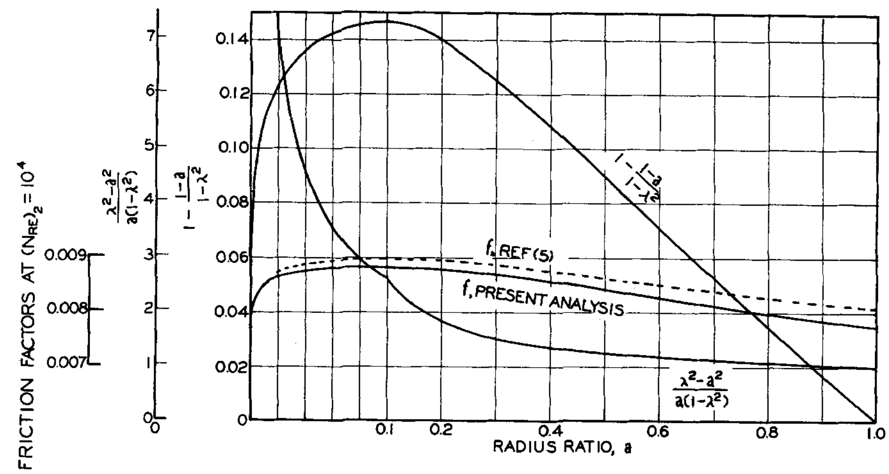

Fig. 2. Plots of various functions.

$$(N_{Re})_2 = \frac{2(r^2_2 - r^2_{max})U}{r_2 \nu}$$
$$= \frac{(1-\lambda^2)2\, r_2 U}{\nu} \qquad (10)$$

Equation (9) is the same as the Nikuradse (8) friction factor equation for smooth circular tubes and is closely approximated by the following less-complicated relationships:

$$f_2 = 0.046\ (N_{Re})_2^{-0.2} \qquad (11)$$
$$f_2 = 0.079\ (N_{Re})_2^{-0.25} \qquad (12)$$

On the basis of the definition of friction factors given in Equations (7) and (8) one can now apply Equation (1) to the inner and outer walls of the annulus as follows:

$$(j_H)_1 = \left(\frac{h_1}{C_pG}\right)\left(\frac{C_p\mu}{k}\right)^{2/3} = \frac{f_1}{2} \qquad (13)$$

$$(j_H)_2 = \left(\frac{h_2}{C_pG}\right)\left(\frac{C_p\mu}{k}\right)^{2/3} = \frac{f_2}{2} \qquad (14)$$

On the basis of definitions given previously it can be shown that

$$f = f_2\left(\frac{1-a}{1-\lambda^2}\right) \qquad (15)$$

$$N_{Re} = (N_{Re})_2\left(\frac{1-a}{1-\lambda^2}\right) \qquad (16)$$

$$\frac{f_1}{f_2} = \frac{\tau_1}{\tau_2} = \frac{\lambda^2 - a^2}{a(1-\lambda^2)} \qquad (17)$$

The relationships given in Equations (15), (16), and (17) give information concerning the way that the friction factors f_1 and f_2 vary as the radius ratio a varies from 0 (circular tube) to 1 (infinite parallel planes). This involves the manner in which the functions $(1-a)/(1-\lambda^2)$ and $(\lambda^2 - a^2)/a(1-\lambda^2)$ vary. Values of these functions are plotted in Figure 2. It is worthwhile to note that the function $(1-a)/(1-\lambda^2)$ has a value of unity at both $a = 0$ and $a = 1$, whereas $(\lambda^2 - a^2)/a(1-\lambda^2)$ is infinite at $a = 0$ and becomes unity at $a = 1$.

The friction factor is plotted as a function of the radius ratio at constant $(N_{Re})_2 = 10^4$ in Figure 2. The physical situation represented by this plot is correct. At constant $(N_{Re})_2$, f_2 is constant, while f_1 is a function of the radius ratio and becomes infinite as $a \to 0$. This does not mean that friction losses become infinite, since as $a \to 0$ the area of the inner tube approaches zero, and in the limit an infinite shear stress acts upon zero area. The friction factor f represents the combined effect of f_1 and f_2. It is seen that $f = f_2$ at $a = 0$ (circular tube) and at $a = 1$ (parallel planes) and goes through a maximum value at $a = 0.11$. These results are in good agreement with those recently presented by Meter and Bird (5). The broken line in Figure 2 represents the predictions of these workers. It is seen that their friction factors are up to 5% greater than those given by the solid curve, but the shape of the two lines is identical.

The j factor for both the inner and outer walls are plotted in Figure 3. The solid curve represents a plot of either

$$(j_H)_2 \text{ vs. } N_{Re}/\left(\frac{1-a}{1-\lambda^2}\right)$$

or

$$(j_H)_1/\left(\frac{\lambda^2 - a^2}{a(1-\lambda^2)}\right)$$
$$\text{vs. } N_{Re}/\left(\frac{1-a}{1-\lambda^2}\right)$$

This curve may be used to obtain either the inner or outer j factor for an annular duct of any radius ratio and Reynolds number $N_{Re} = 2bU/\nu$. (Note that in this definition of the Reynolds number b represents the thickness of the annular gap which becomes the distance between parallel planes when the radius ratio becomes unity.)

The curve is plotted from Equation (9). If one uses Equation (11), an explicit relation for $(j_H)_1$ and $(j_H)_2$ may be obtained. Thus

$$(j_H)_2 = \frac{f_2}{2} = 0.023\ (N_{Re})_2^{-0.2}$$
$$= 0.023\, N_{Re}^{-0.2}\left(\frac{1-a}{1-\lambda^2}\right)^{0.2} \qquad (18)$$

$$(j_H)_1 = \frac{f_1}{2} = \frac{\lambda^2 - a^2}{a(1-\lambda^2)}\ \frac{f_2}{2}$$
$$= 0.023\, N_{Re}^{-0.2}\left(\frac{1-a}{1-\lambda^2}\right)^{0.2}\left(\frac{\lambda^2 - a^2}{a(1-\lambda^2)}\right) \qquad (19)$$

Use of Equations (18) and (19) gives values of j factor which are within 6% of the curve in Figure 3 at $N_{Re}/$

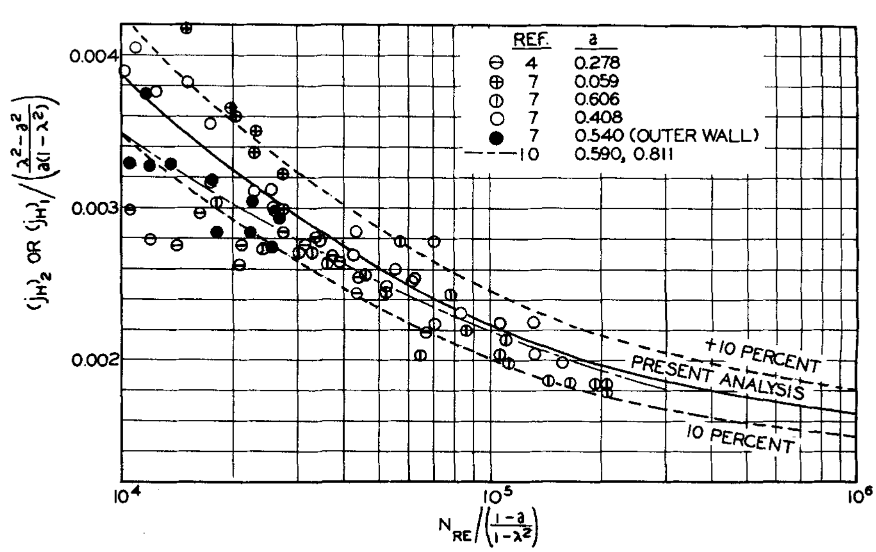

Fig. 3. *j* factor-Reynolds number plot.

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INFORMATION RETRIEVAL

Key Words: Path Number-6, Polarity Number-6, Normal Boiling Point-8, Critical Temperature-8, Critical Pressure-8, Critical Volume-8, Critical Compressibility Factor-8.

Abstract: An approach which utilizes the polarity number and the path number for the calculation of normal boiling points of saturated aliphatic hydrocarbons has been extended to include the calculation of the critical constants of these substances. Relationships were first established for the critical properties of the normal paraffins. The differences between the critical properties of a normal paraffin and of an isomeric aliphatic hydrocarbon containing the same number of carbon atoms were then related to the corresponding differences in the path number and polarity number. This method was found to produce reliable values of the properties investigated.

Reference: Stiel, Leonard I., and George Thodos, **A.I.Ch.E. Journal, 8,** No. 4, p. 536 (September, 1962).

Key Words: Porous Vycor-5, Temperature Gradient-6, Surface Flow-7, Hydrogen-9, Helium-9, Nitrogen-9, Argon-9, Ethylene-9, Propylene-9, Adsorption-8.

Abstract: The isothermal and isobaric flow of hydrogen, helium, nitrogen, argon, ethylene, and propylene through microporous Vycor glass were investigated under conditions such that gas-phase flow occurred by Knudsen diffusion. Data for the non-adsorbed gases are correlated by existing relationships. A relation is developed to describe the flow of the adsorbed gases.

Reference: Gilliland, E. R., R. F. Baddour, and H. H. Engel, **A.I.Ch.E. Journal, 8,** No. 4, p. 539 (September, 1962).

Key Words: Methane-1, Carbon Dioxide-1, Nitrogen-3, Phase Behavior-8, Solubility-8, Purification-9, Natural Gas-9, Pressure-6, Temperature-6, Composition-7.

Abstract: The solid-liquid-vapor phase behavior of the methane-carbon dioxide system is described. Pressure-temperature measurements along the S-L-V locus are presented from the triple point of carbon dioxide to —284°F. Vapor phase compositions along the S-L-V locus have been determined from 0.1 to 12% carbon dioxide. Liquid phase compositions have been determined from 0.16 to 20% carbon dioxide. Extra polation of earlier data gives consistent values over the remaining composition ranges. The effect of a small amount of nitrogen on the S-L-V locus is reported.

Reference: Davis, J. A., Newell, Rodewald, and Fred Kurata, **A.I.Ch.E. Journal, 8,** No. 4, p. 546 (September, 1962).

Key Words: Adsorption-5, Methane-1, Hydrogen-1, Silica Gel-4, Diffusion-8, Mass Transfer-8, Flow-, Fixed-, Beds-, Rate-, Internal-.

Abstract: The adsorption of mixtures of methane and hydrogen on fixed beds of silica gel at —115°F. and 1 atm. has been studied. The rate of internal diffusion can be correlated by a convenient mathematical expression describing the rate of transfer in the adsorbent particle. In this work K_p was found to be dependent on gas flow rate.

Reference: Campbell, M. Larry, and Lawrence N. Canjar, **A.I.Ch.E. Journal, 8,** No. 4, p. 549 (September, 1962).

Key Words: Predicting-8, Estimating-8, Critical Temperature-8, Properties (Characteristics)-8, Physical Properties-8, Thermodynamics-9, Hydrocarbons-9, Mixtures-9, Methane-9, Correlations-10.

Abstract: The available data on critical temperatures of hydrocarbon mixtures have been reviewed in order to develop a direct and accurate prediction method. Two correlations are presented for methane-free and for methane-containing mixtures. They can be used to estimate the critical temperatures of mixtures containing from two to five components.

Reference: Grieves, Robert B., and George Thodos, **A.I.Ch.E. Journal, 8,** No. 4, p. 559 (September, 1962).

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$\left(\frac{1-a}{1-\lambda^2}\right) = 10^4$ and 10^7 and considerably closer to the curve at intermediate Reynolds numbers.

Several empirical equations have been proposed for predicting heat transfer coefficients in annuli, and attempts have been made to account for the radius ratio. An equation commonly recommended is of the form

$$(j_H)_1 = C\, N_{Re}^{-0.20}\, (a)^{-n} \qquad (20)$$

Monrad and Pelton (7) give $n = 0.53$ on the basis of the variation of the two factors on the right side of Equation (20). Weigand (*11*) gives $n = 0.45$, while Stein, et al. (*10*) give $n = 0.50$. Values of C range from 0.020 to 0.023. All equations of the form of Equation (20) agree with the solid line of Figure 3 quite well, except at extreme values of the radius ratio.

Experimental heat transfer data obtained by various workers (*4, 7, 10*) at values of a between 0.058 and 0.811 are plotted in Figure 3. The majority of the points lie within ± 10% of the solid line, although there is somewhat more scatter in the vicinity of $(N_{Re})_2 = 10^4$. The solid curve therefore may be used to predict j factors for annuli of any radius ratio.

The results described herein are summarized as follows:

1. Previous workers (9) have shown that Equation (9) is satisfactory for predicting values of f_2 from which f_1 and f may be calculated from Equations (15) and (17).

2. Employing Colburn's analogy one may obtain the j factor at each wall of the annulus from Figure 3. This procedure is restricted to fluids with Prandtl numbers greater than 0.6.

3. The single curve in Figure 3 may be used for annuli of any radius ratio from circular tube ($a = 0$) to parallel planes ($a = 1$).

NOTATION

- a = radius ratio, r_1/r_2
- b = gap between inner and outer tubes of annulus, ft.
- C_p = heat capacity at constant pressure, B.t.u./lb.$_m$ °F.
- f = Fanning friction factor
- G = mass velocity, lb.$_m$/hr. sq.ft.
- g_c = gravitational conversion factor, lb.$_m$ ft./lb.$_f$ sec.^{2}
- h = heat transfer coefficient, B.t.u./hr. sq.ft. °F.
- j_H = j factor for heat transfer
- k = thermal conductivity, B.t.u./sec. sq.ft. °F./ft.
- N_{Re} = Reynolds number defined by Equation (5)

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$(N_{Re})_2$ = Reynolds number defined by Equation (10)

$-(dP_f/dx)$ = pressure gradient due to friction, lb.$_f$/cu.ft.

r = radius, ft.

r_{max} = radius at point of maximum velocity, ft.

U = average velocity of fluid in duct, ft./sec.

Greek Letters

λ = r_{max}/r_2

μ = viscosity, lb.$_m$/ft. sec.

ν = kinematic viscosity, sq.ft./sec.

ρ = density, lb.$_m$/cu.ft.

τ_w = shear stress at the wall, lb.$_f$/sq.ft.

Subscripts 1 and 2 refer to inner and outer wall of annulus, respectively.

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INFORMATION RETRIEVAL

Key Words: Heat Transfer-8, Fluid Flow-8, Flow-8, Non-Newtonian-8, Liquids-9, Fluids-9, Power Law-, Rheology-9, Viscosity-9, Properties (Characteristics)-9, Physical Properties-9, Boundary Layer-9, Laminar Flow-9, Temperature Distribution-9, Carboxymethyl-Cellulose-1, Ethers-1, Water-5, Water Tunnel-10, Cylinders-10.

Abstract: Heat transfer from a uniformly heated cylinder to a power-law non-Newtonian fluid has been studied in a water tunnel, with a dilute solution of carboxymethyl cellulose (CMC) in water. The experiments were restricted to CMC solutions which follow the power law and which, to all appearances, do not show viscoelastic effects. The experiments were performed under laminar boundary-layer flow conditions with the axis of the cylinder normal to the flow. Experimentally measured temperature distributions at the surface of the cylinder are compared with the results of a theoretical analysis which only requires, for each fluid, the independent evaluation of the rheological parameters of the power law.

Reference: Shah, M. J., E. E. Petersen, and Andreas Acrivos, **A.I.Ch.E. Journal, 8,** No. 4, p. 551 (September, 1962).